

模糊邏輯與不確定性處理

產業研發碩士專班課程

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本週主題

1. 不確定性與模糊邏輯
2. 模糊邏輯運算
3. 模糊邏輯應用

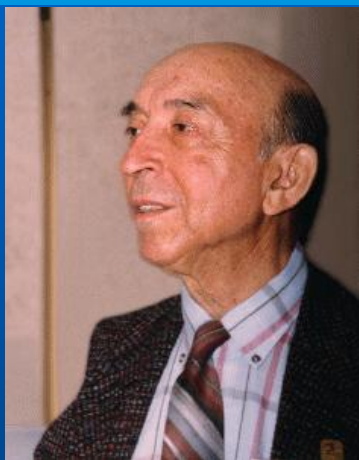


不確定性與模糊邏輯

Uncertainty and Fuzzy Logic



Lotfi Zadeh—模糊邏輯之父



- 模糊邏輯之父。
- Berkeley大學Soft Computing研究院
- 1965年發表模糊集合理論，對自然語言中的不確定性資訊從事建模。



不確定性的種類

1. 決定性(**Deterministic**)：
不確定性是一個已知實數與其近似值之間差異量(誤差)
2. 區間(**Interval**)：
不確定性是一個測量的區間
3. 機率(**Probabilistic**)：
不確定性是一個機率密度函數
4. 模糊(**Fuzzy**)：
不確定性是一個模糊歸屬函數
5. 含混(**Ambiguity**)：
不確定性是一對多的關係



布林邏輯與模糊邏輯

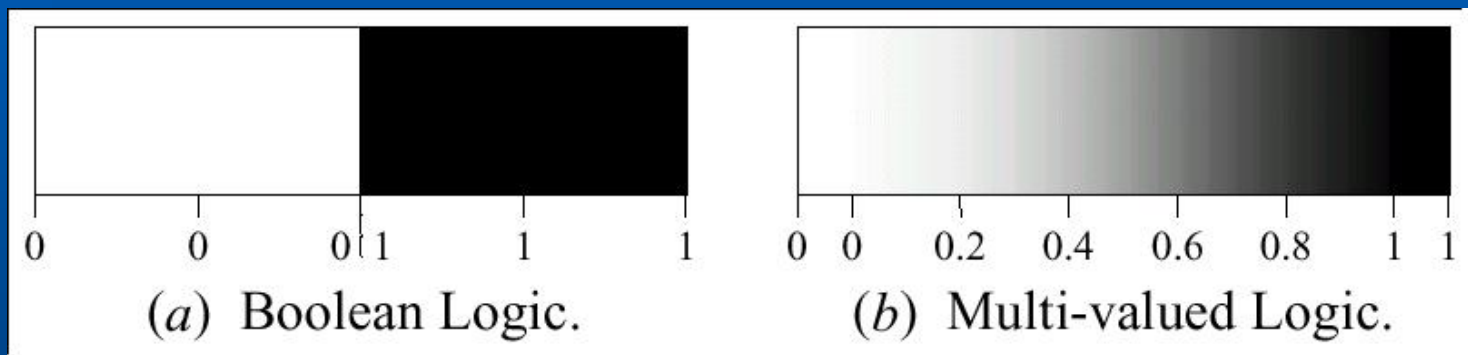
- 妻子：*Do you love me?*
- 丈夫：*Yes.*(布林邏輯)
- 妻子：*How much?* (模糊邏輯)



不確定性的種類

布林邏輯(Boolean Logic)：二值

模糊邏輯(Fuzzy Logic)：多值



布林邏輯： $\{\text{真}, \text{偽}\}$ ； $\{0, 1\}$

模糊邏輯：部分為真(部分為偽)



自然語言的模糊邏輯表示

模糊邏輯處理變數的歸屬度(membership)和確定度(degrees of certainty)：

- 溫度——“溫度**很高**”
- 電壓——“電壓**有點偏低**”
- 速度——“速度**非常慢**”



自然語言的模糊邏輯表示

Name	Height	Hypothesis	
		Deg of Membership (Crisp)	Deg of Membership (Fuzzy)
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00



模糊邏輯的運算

若有模糊邏輯陳述如下：

X is LOW

則

X is LOW and Y is HIGH or (not Z is MEDIUM)

應如何解釋？

$\text{truth}(\text{not } x) = 1.0 - \text{truth}(x)$

$\text{truth}(x \text{ and } y) = \text{minimum}(\text{truth}(x), \text{truth}(y))$

$\text{truth}(x \text{ or } y) = \text{maximum}(\text{truth}(x), \text{truth}(y))$



模糊集合

- 一個模糊集合是一個具有模糊邊界的集合

- 二元集合理論：

$$f_A(x):X \rightarrow \{0,1\}, \text{ where } f_A(x) =$$

$$\begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

- 模糊集合：

$$\mu_A(x):X \rightarrow \{0,1\},$$

where $\mu_A(x) = 1$, if x is totally in A ;

$\mu_A(x) = 0$, if x is not in A ;

$0 < \mu_A(x) < 1$, if x is partly in A



模糊集合

- $\mu_A(x)$ 是歸屬函數 (membership function)
- 歸屬函數值介於 0 和 1 之間
- 此一值表示元素 x 在集合 A 的歸屬程度 (degree of membership)

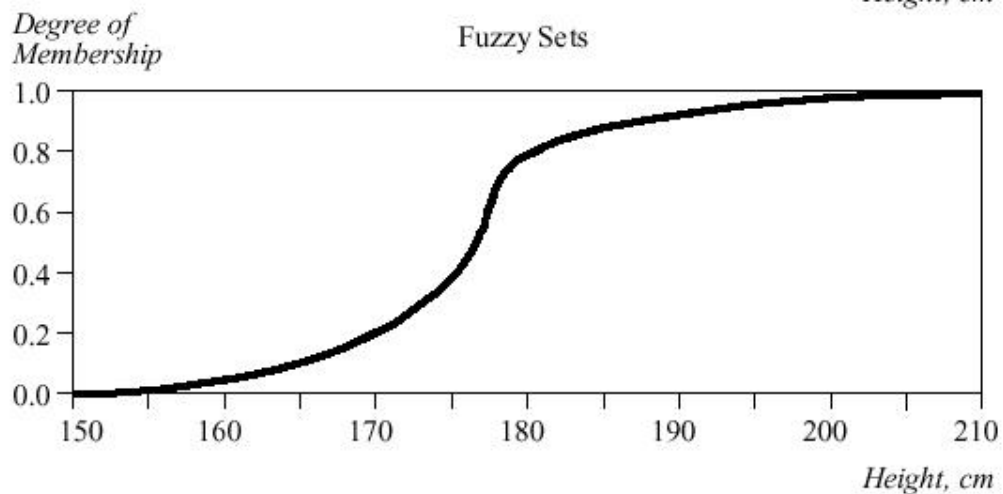
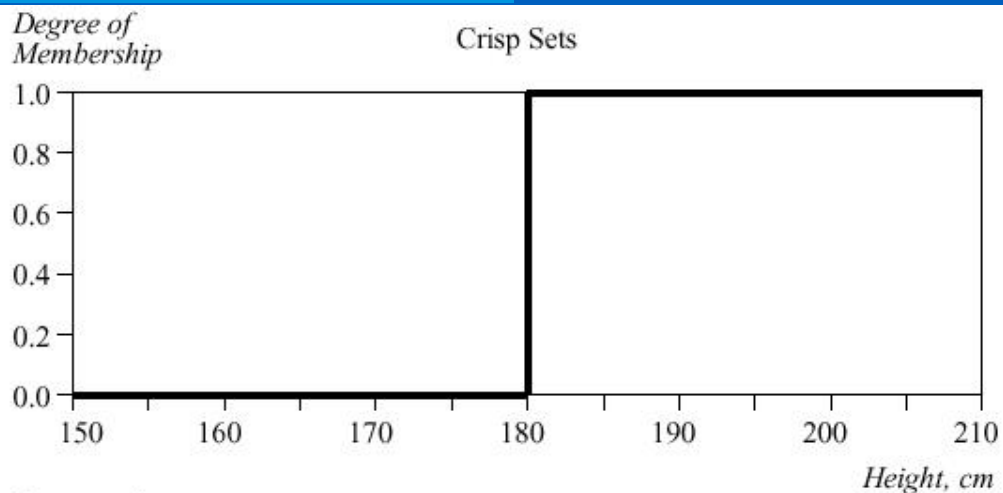


身高的二元集合和模糊集合

Name	Height, cm	Degree of Membership	
		<i>Crisp</i>	<i>Fuzzy</i>
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00

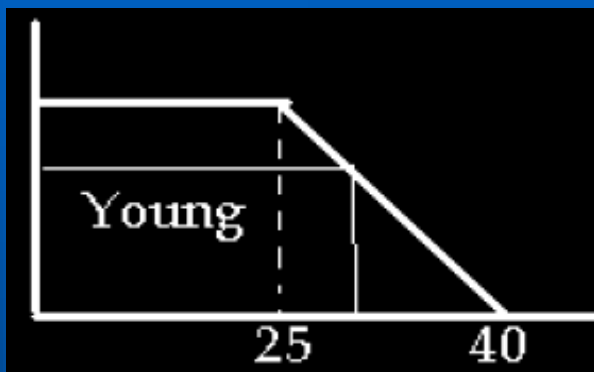


身高的二元集合和模糊集合





Young的模糊集合及歸屬度計算



$$\mu_{Young}(x) = \begin{cases} 1; & x \leq 25 \\ \frac{40-x}{15}; & 25 < x \leq 40 \\ 0; & elsewhere \end{cases}$$

如果 x 是22歲，則其屬於集合Young的歸屬度是1。如果 x 是30歲，則其屬於集合Young的歸屬度是0.666。



模糊邏輯運算

Operations for Fuzzy Logic



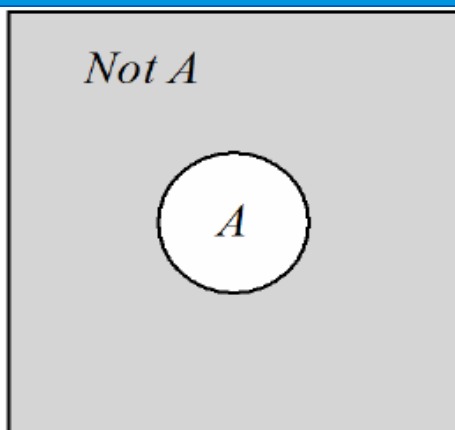
集合運算(聯集)

二元邏輯：
 $x \in A \cup B$
 $x \in C \text{ iff } x \in A \text{ or } x \in B$

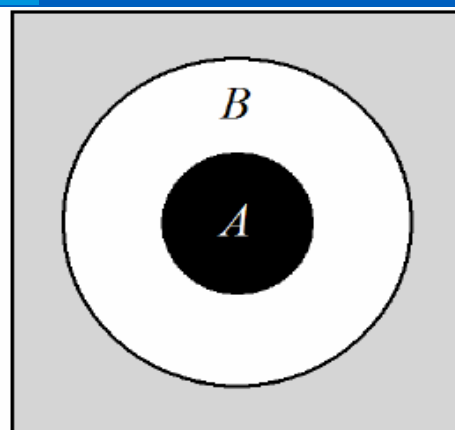
模糊邏輯：
 $\mu_A(x) = 0.6, \mu_B(x) = 0.4$
 $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$



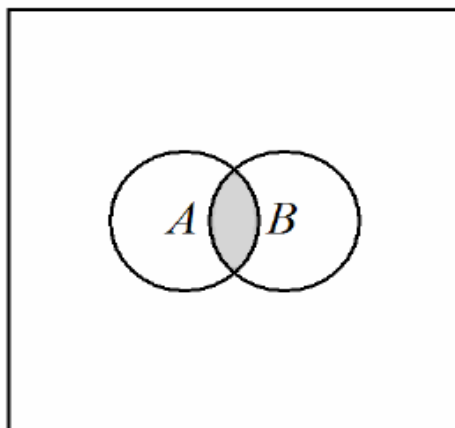
二元集合運算



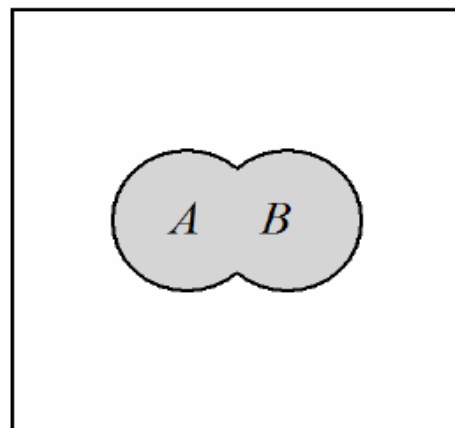
Complement



Containment



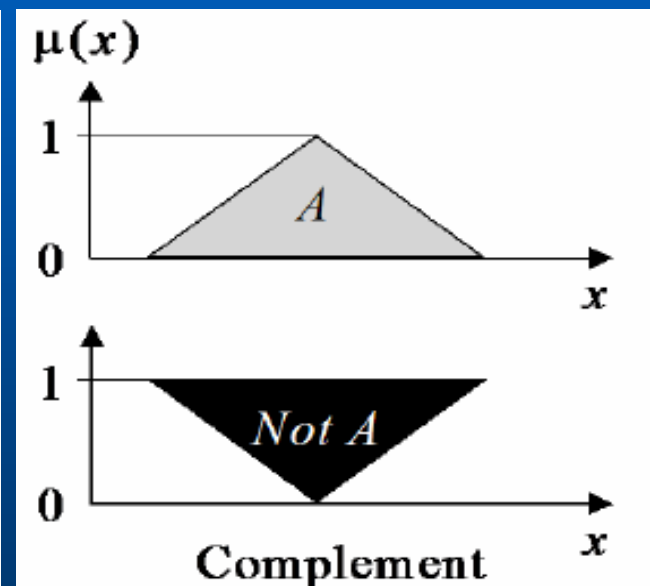
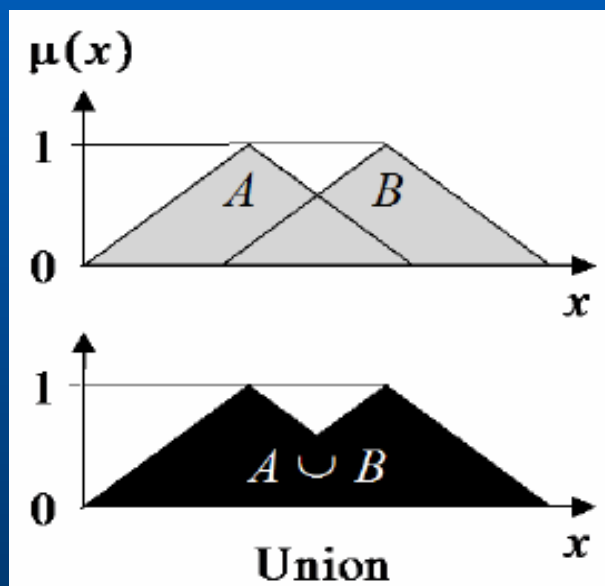
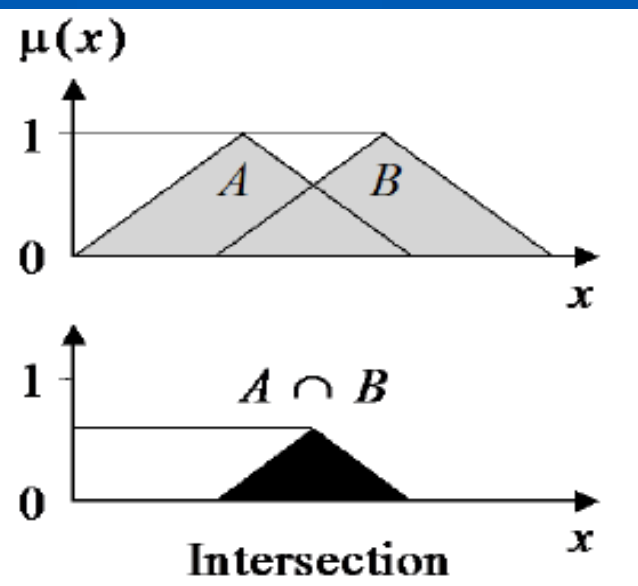
Intersection



Union



模糊集合運算

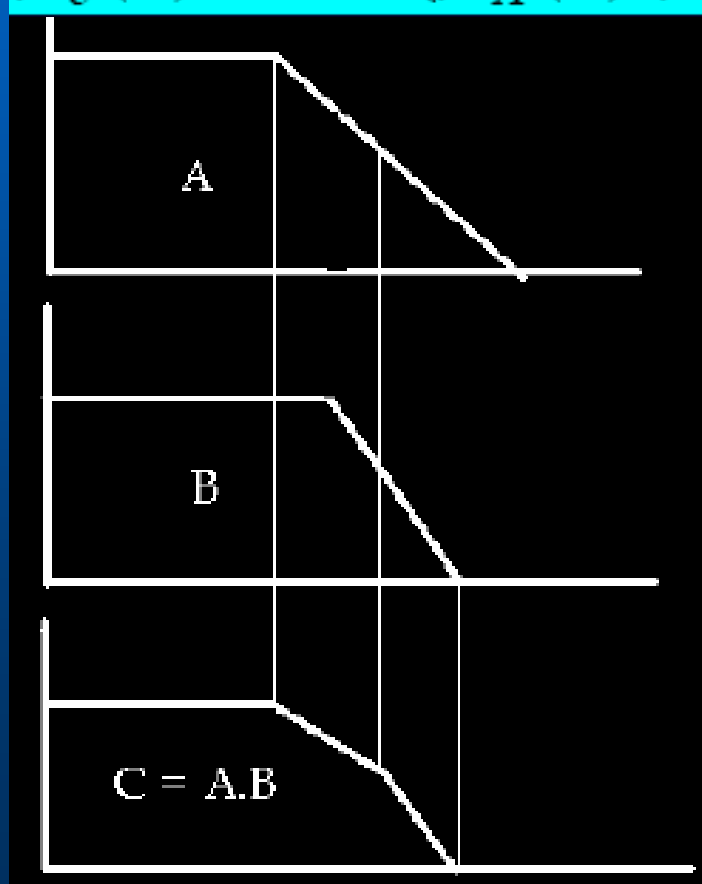
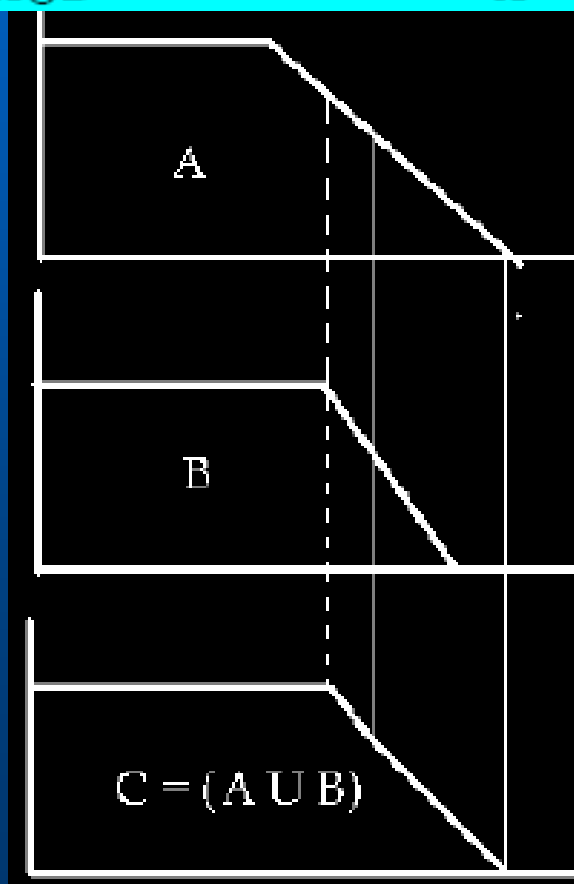




模糊集合運算

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

$$\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$$





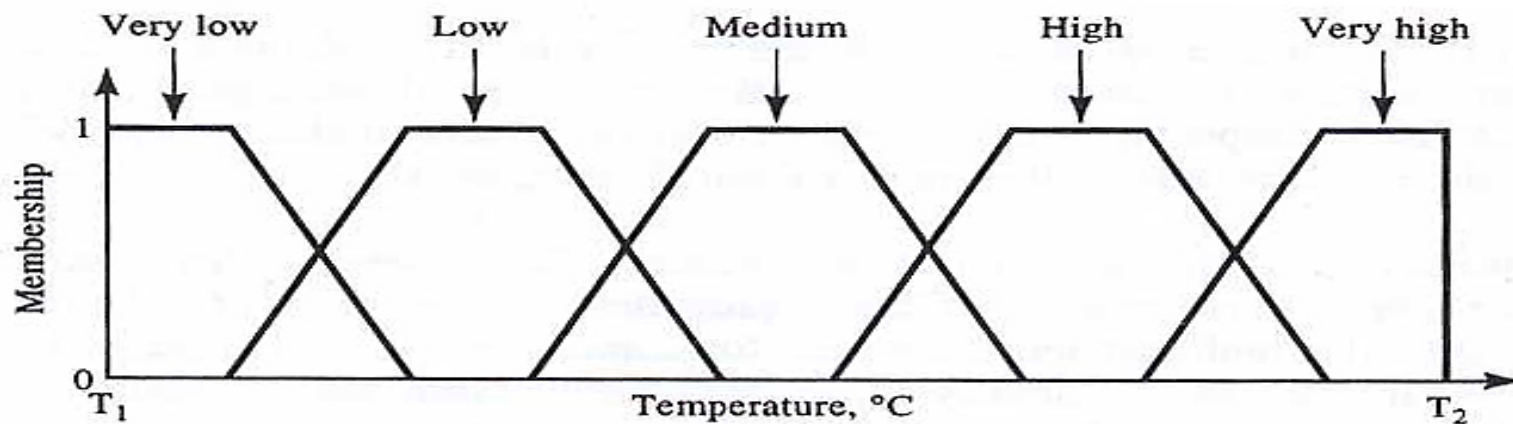
語言變數與模糊集合

語言變數：

Temperature

模糊集合：

Temperature = { Very low, Low, Medium, High, Very high }

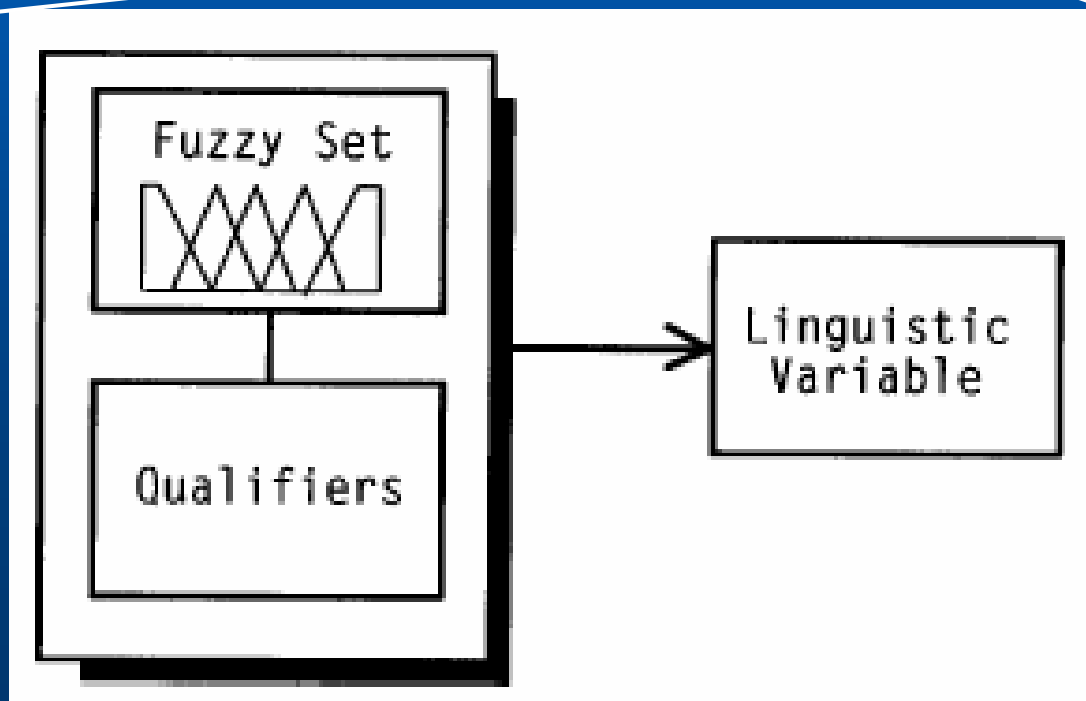




語言的計算

if **project.duration** is **LONG**
then the **completion.risk** is **INCREASED**;

語言變數



模糊集合

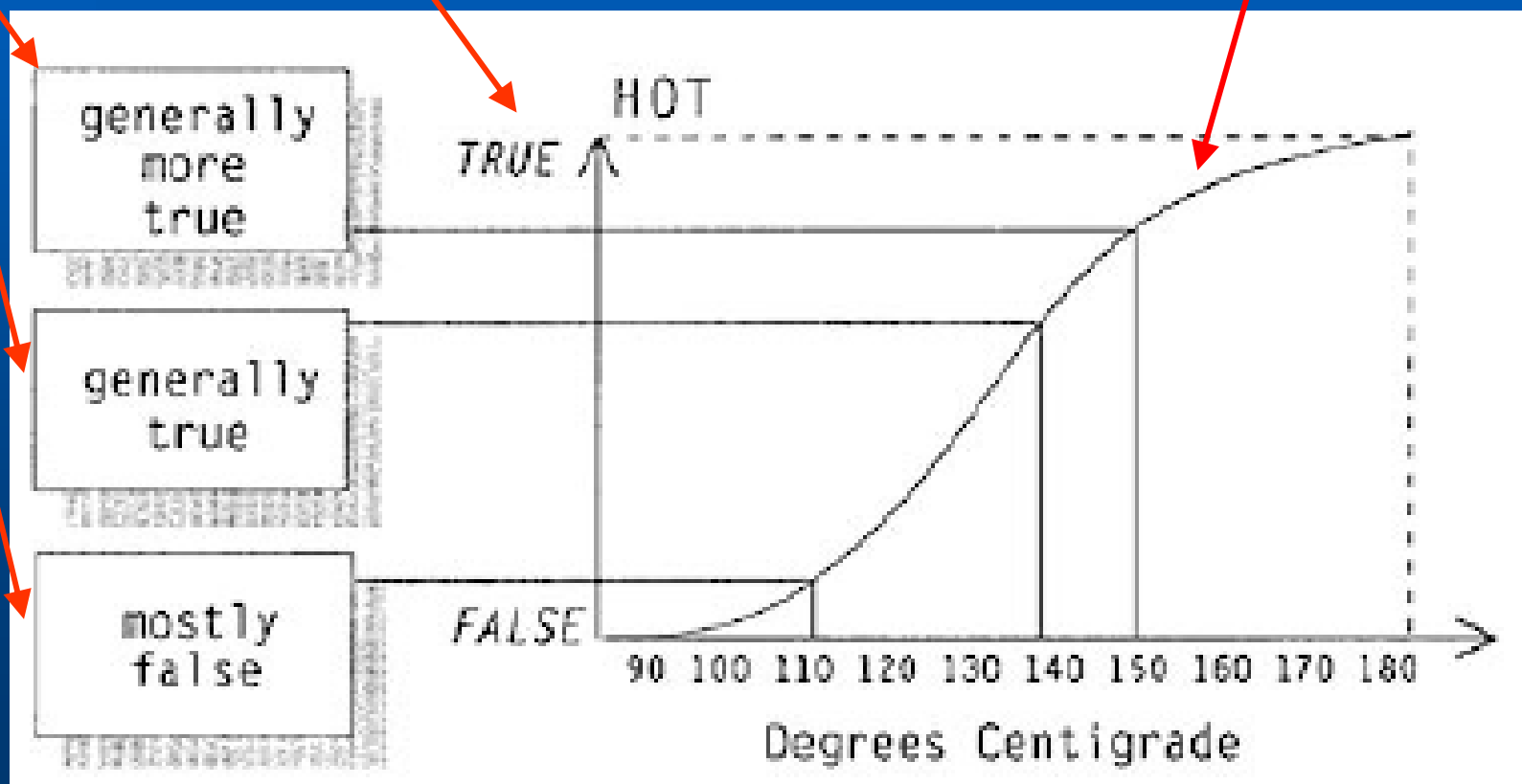


語言變數的模糊評估

Qualification

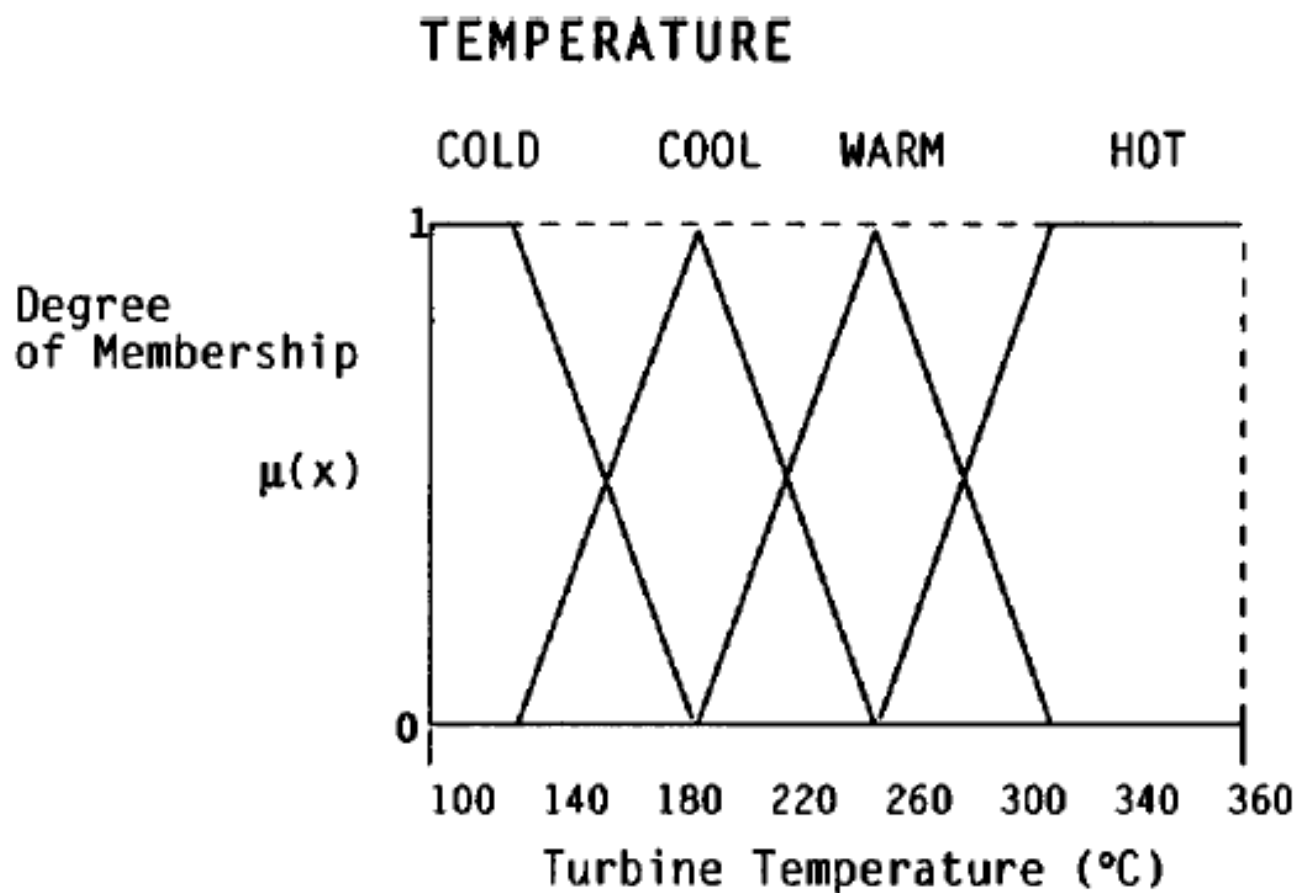
歸屬度

HOT模糊集合





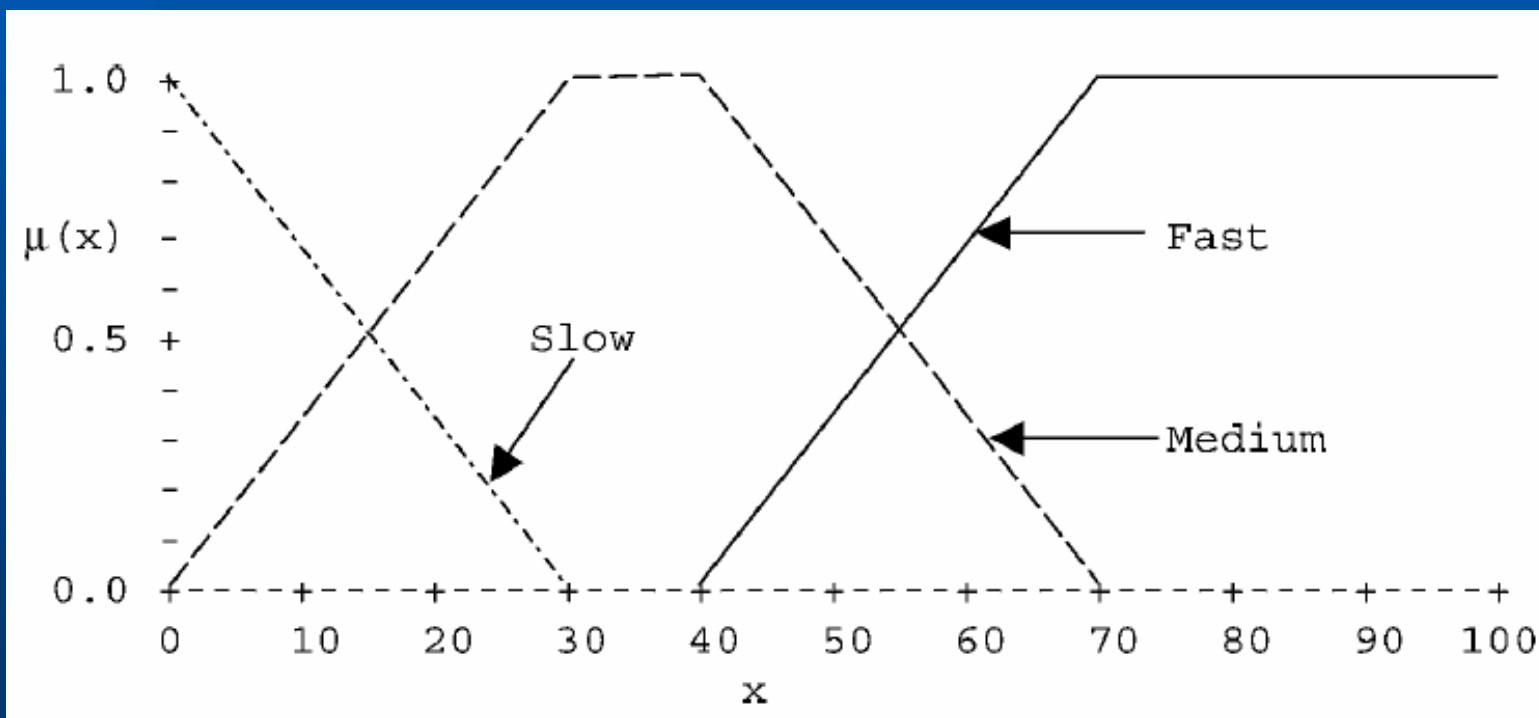
語言變數的所有模糊集合





將數值資料模糊化

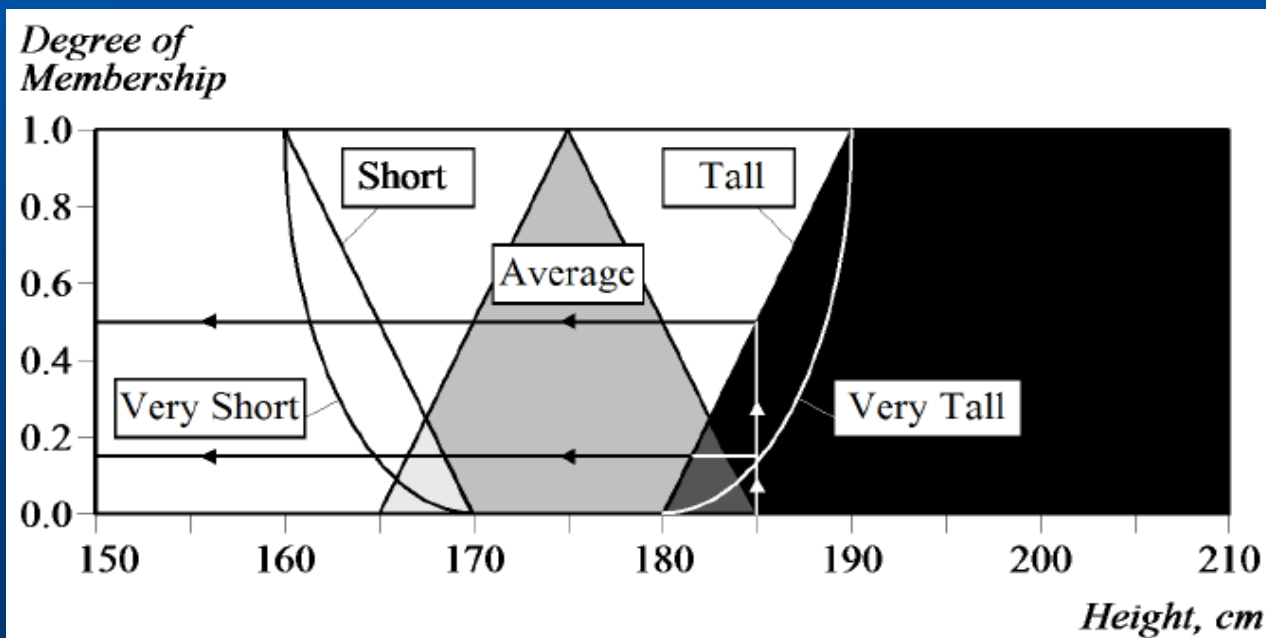
$$\text{Speed} = \left\{ \frac{\mu_1}{\text{small}}, \frac{\mu_2}{\text{medium}}, \frac{\mu_3}{\text{large}} \right\}$$









模糊集合的修飾語(qualifier)

- 一個語言變數伴隨一個模糊集合的修飾語稱之為Hedge
- Hedge是一個改變模糊集合形狀的運算子





模糊集合的修飾語(qualifier)

A little	$[\mu_A(x)]^{1.3}$	
Very	$[\mu_A(x)]^2$	
Extremely	$[\mu_A(x)]^3$	
More or less	$\sqrt{\mu_A(x)}$	



模糊邏輯應用

Applications of Fuzzy Logic



模糊規則(Fuzzy Rule)

一個模糊規則如下 IF-THEN 條件陳述：

IF x is A

THEN y is B

A, B 是模糊集合



Crisp規則與模糊規則

傳統專家系統的二元規則：

IF score is > 80

THEN select

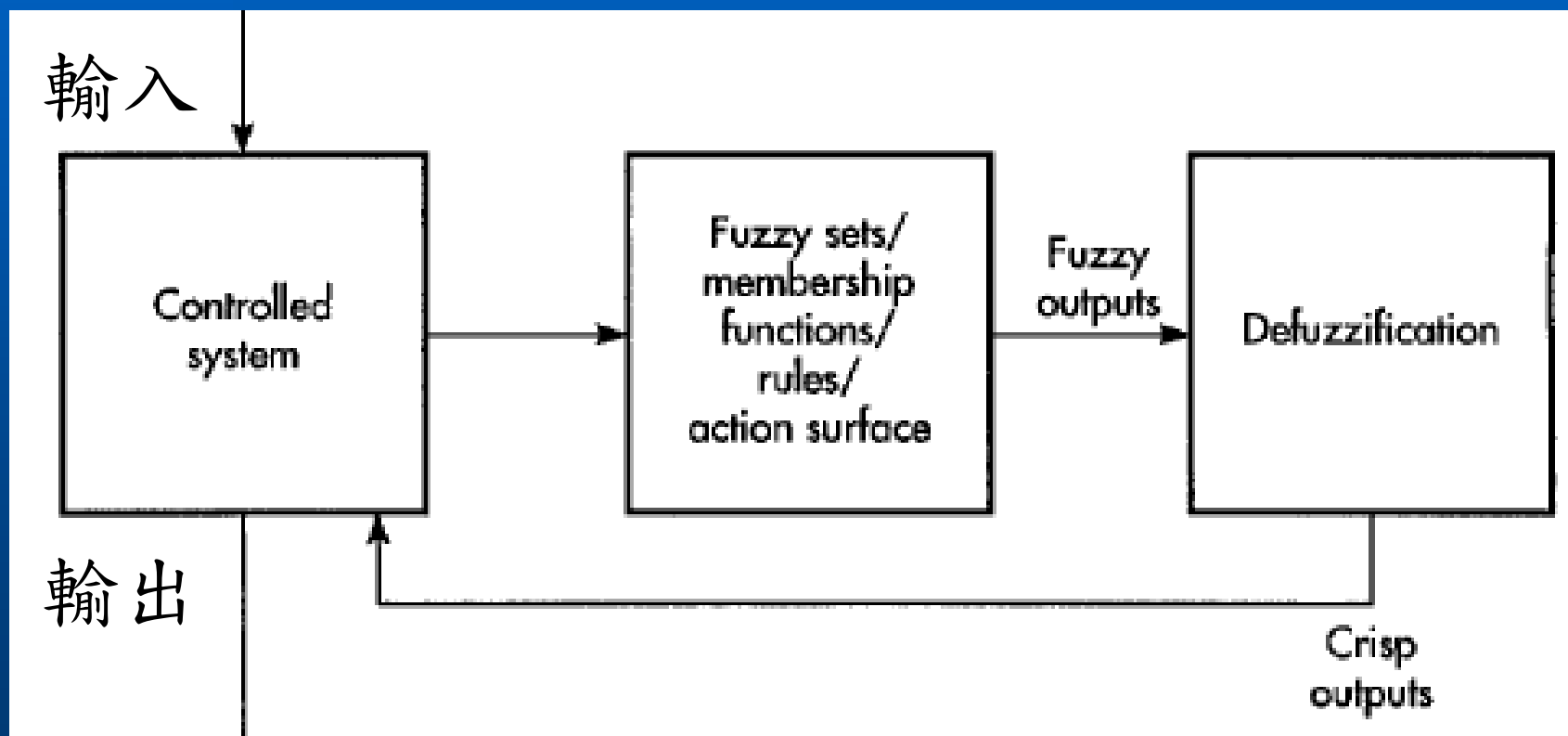
模糊專家規則：

IF score is high

THEN select

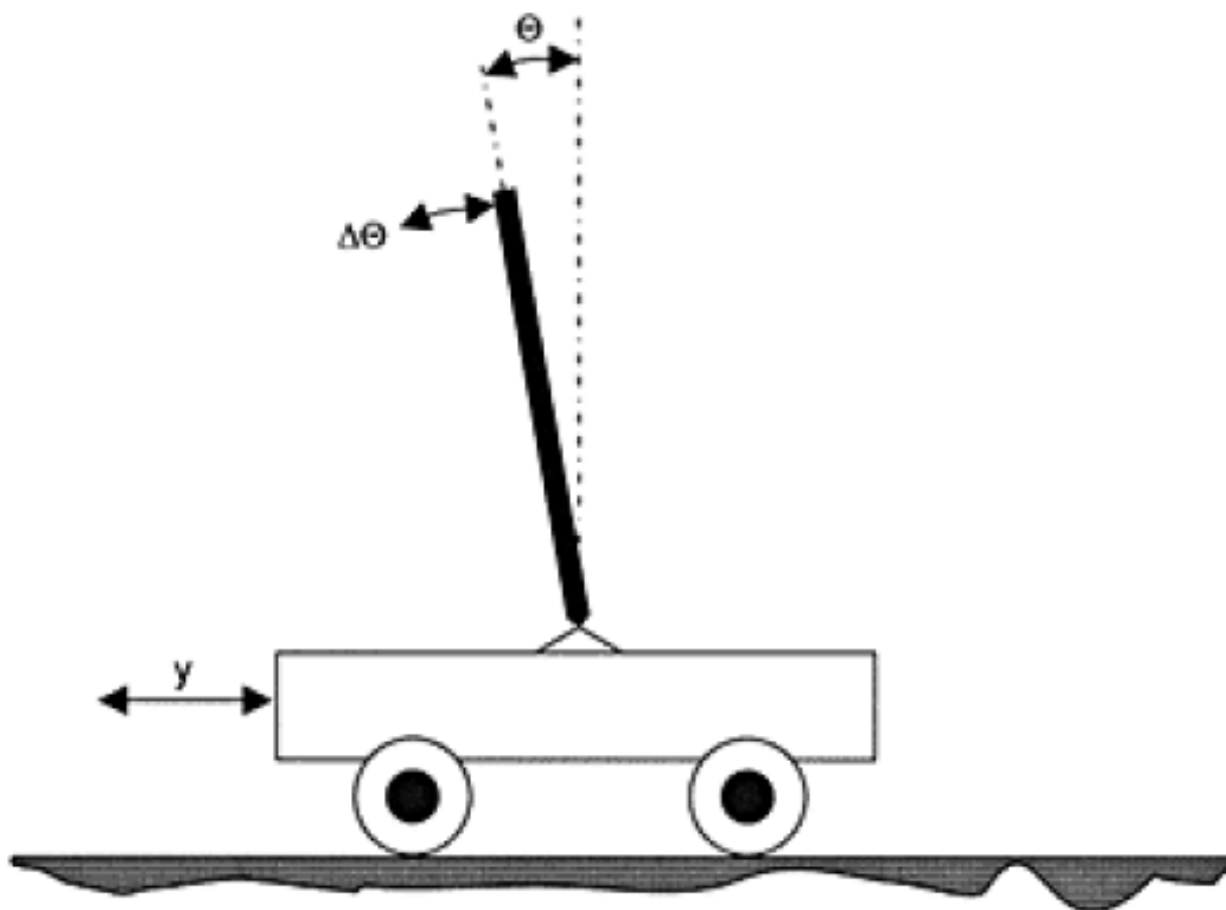


模糊控制





倒單擺的模糊控制





倒單擺的模糊控制

語言變數：

夾角 angle Θ

角速度 $\Delta \Theta$

施力 y

五個模糊集合：

(1) positive medium (PM);

(2) positive small (PS);

(3) zero (ZE);

(4) negative small (NS);

(5) negative medium (NM)



模糊控制規則

IF θ is PM AND $\Delta\theta$ is ZR, THEN y is PM.

IF θ is PS AND $\Delta\theta$ is PS, THEN y is PS,

IF θ is PS AND $\Delta\theta$ is NS, THEN y is ZR,

IF θ is NM AND $\Delta\theta$ is ZR, THEN y is NM,

IF θ is NS AND $\Delta\theta$ is NS, THEN y is NS,

IF θ is NS AND $\Delta\theta$ is PS, THEN y is ZR,

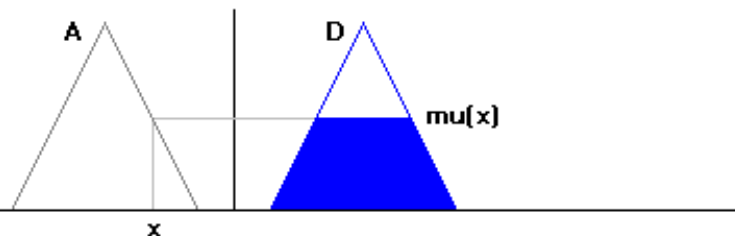
IF θ is ZR AND $\Delta\theta$ is ZR, THEN y is ZR.



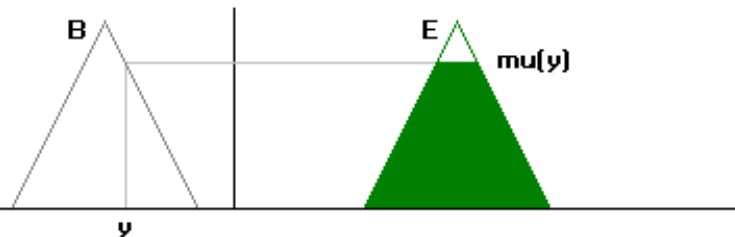
模糊控制

GVG/PD/1.0

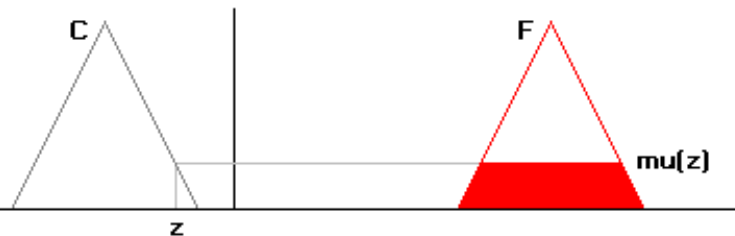
rule 1: IF x IS A THEN n IS D:



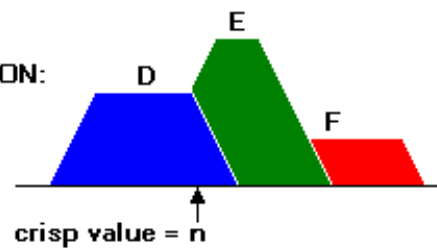
rule 2: IF y IS B THEN n IS E:



rule 3: IF z IS C THEN n IS F:



DEFUZZIFICATION:

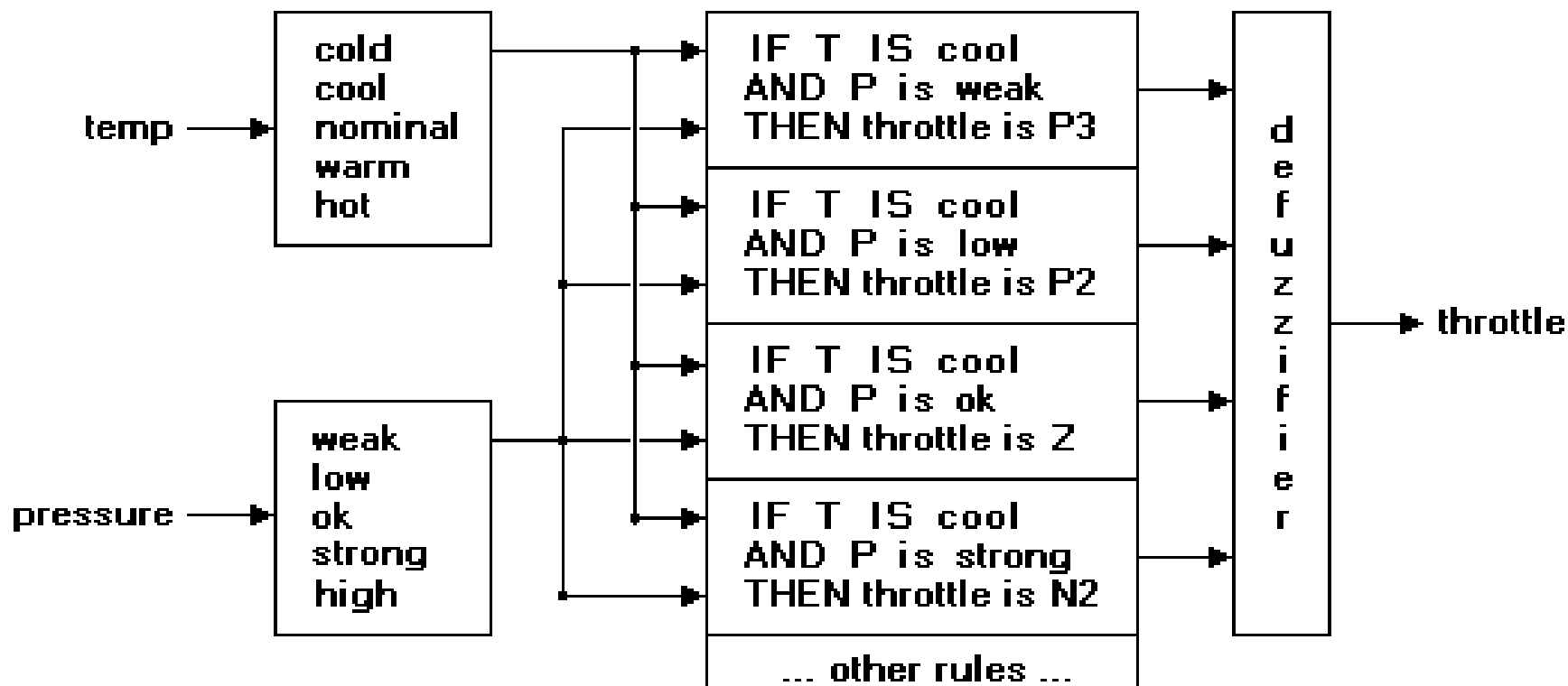


CENTROID DEFUZZIFICATION
USING MAX-MIN INFERENCE



模糊控制

BVG/PD/1.0





每一語言變數的模糊集合

N3: Large negative.

N2: Medium negative.

N1: Small negative.

Z: Zero.

P1: Small positive.

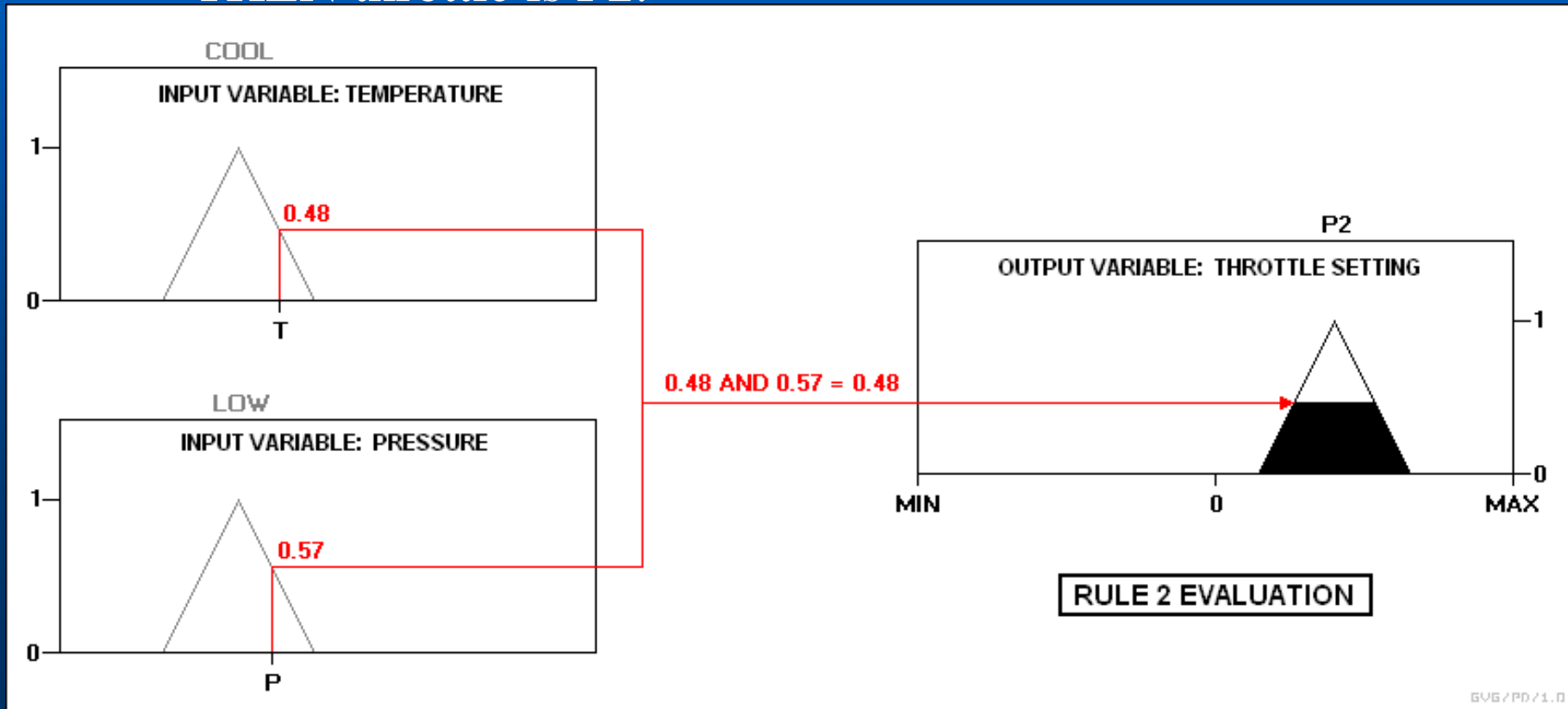
P2: Medium positive.

P3: Large positive.



應用模糊規則

rule 2: IF temperature IS cool AND pressure IS low,
THEN throttle is P2.



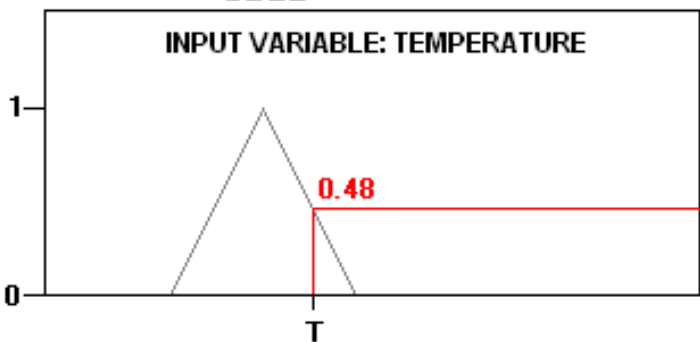


應用模糊規則

rule 3: IF temperature IS cool AND pressure IS ok,
THEN throttle is Z.

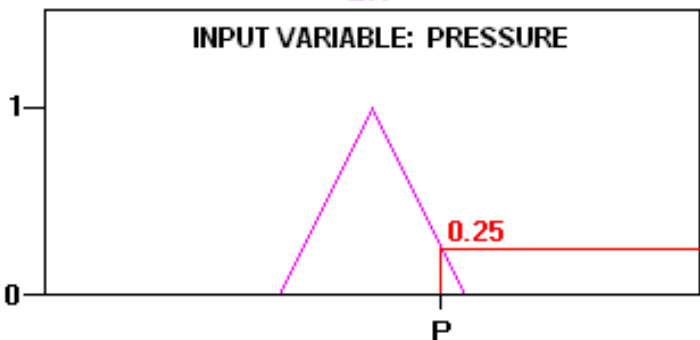
COOL

INPUT VARIABLE: TEMPERATURE



OK

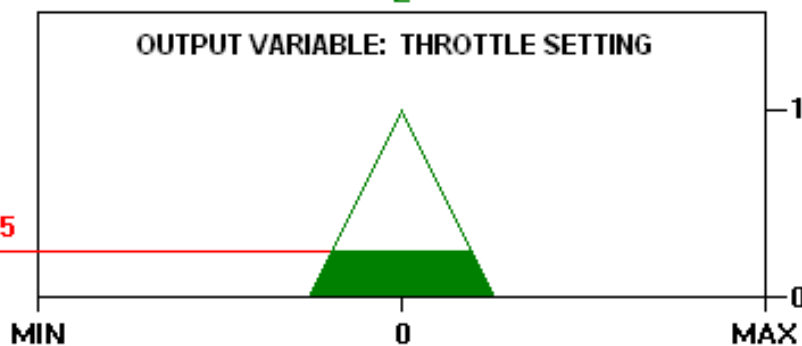
INPUT VARIABLE: PRESSURE



0.48 AND 0.25 = 0.25

Z

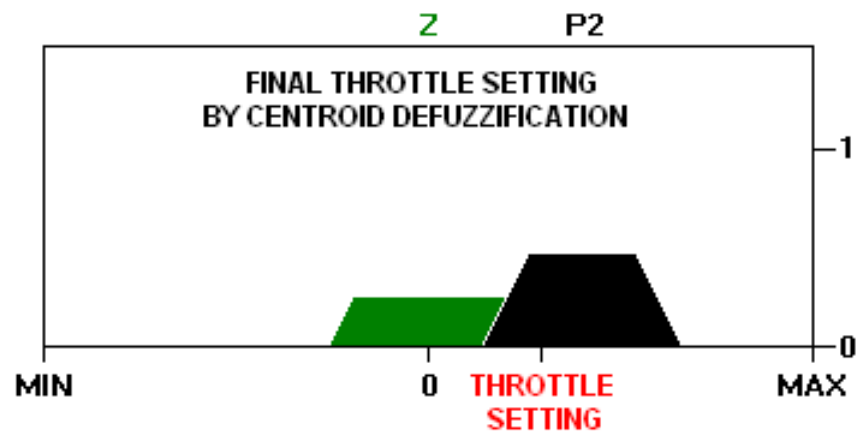
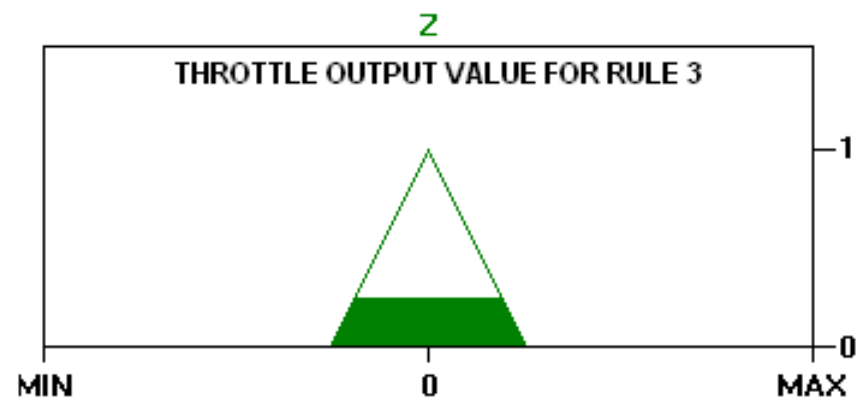
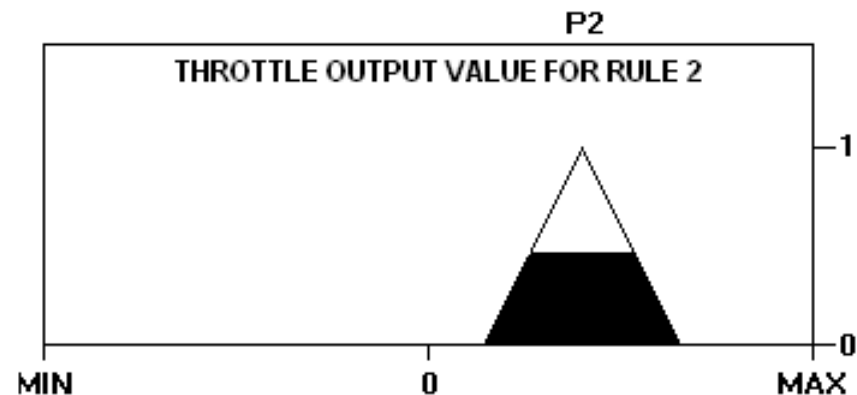
OUTPUT VARIABLE: THROTTLE SETTING



RULE 3 EVALUATION



模糊推論





結語

模糊邏輯模仿人類自然語言的不確定性語義表示，可作為複雜系統中的不確定性資訊建模，並提供有效的運算工具。